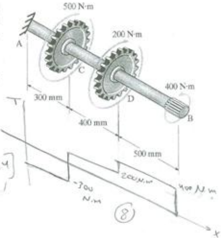
I. 40 points

The gears attached to the steel shaft and the splined end at B are subjected to the torques shown. The shaft is fixed at A. Assume that the shear modulus of elasticity is 80 GPa and the shaft has a diameter of 12 mm:



20 • Determine the angle of twist of twist of end B relative to end A.

(1) $\phi_{B/A} = \Sigma \frac{TL}{GJ}$;

(3) $J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.006)^4 = 2.036\times10^{-9} m^4$ 2.77×10^{-9}

(7) $\phi_{B/A} = \frac{1}{80\times10^9\times2.036\times10^{-9}}\left[-300\times0.3+200\times0.4+400\times0.5\right]$

(6) $\phi_{B/A} = 1.167$ rad (2) 0.5592

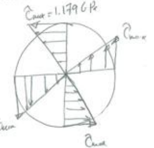
12 • Determine the maximum shear stress in the shaft.

(3) $T_{max} = 400 N.m$

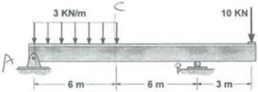
(9) $\tau_{max} = \frac{Tc}{J} = \frac{400\times0.006}{2.036\times10^{-9}} = 1.179\times10^{9}$ Pa $= 1.179$ GPa.

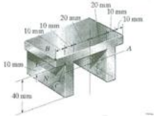
(2) (6) (1) 742.5 MPa

(8) • Draw shear stress distribution on the cross section of the shaft.



II. 40 points





7 • Draw the shear diagram

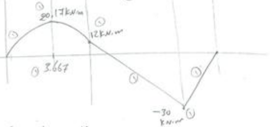
$\circlearrowright \sum M_A = 0$

$\Rightarrow F_B = \frac{10\times15 + 18\times3}{12} = 17\,kN$

$+\uparrow \sum F_y = 0$

$\Rightarrow F_A = (10+18) - 17 = 11\,kN$

7 • Draw the moment diagram

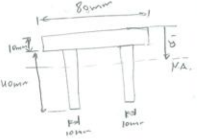


7 • Locate the centroid

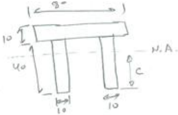
$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = \frac{80\times10\times5 + 2\times40\times10\times(10+20)}{80\times10 + 2\times40\times10}$$

$$= 17.5\ mm$$

from the bottom $\bar{y} = 32.5$ mm.



(II Continued). For the previous figure:



7/ • Find the moment of inertia about the horizontal neutral axis

$I = \frac{1}{12}(80)(10)^3 + (80)(10)(17.5-5)^2 +$

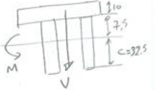
$2\times\left[\frac{1}{12}(10)(40)^3 + 10\times40\times(30-17.5)^2\right]$

$I = 363.3\times10^3\ mm^4 = 363.3\times10^{-9}\ m^4.$

7/ • Determine the maximum flexure and shear stresses

$\sigma_{max} = \frac{M_{max}\times c}{I}$ $\quad M_{max} = 30\,kN \quad c = 50-17.5 = 32.5\,mm$

$= \frac{30\times10^3\times0.0325}{363.3\times10^{-9}} = 2.684\times10^9\ Pa = 2.684\ GPa$ Compression



$\tau_{max} = \frac{V_{max}\,Q}{It} \Rightarrow \quad V_{max} = 11\ kN.$

$Q = 2(10)(32.5)(32.5/2) = 10.56\times10^3\ mm^3 = 10.56\times10^{-6}\ m^3$

$\tau_{max} = \frac{11\times10^3\times10.56\times10^{-6}}{363.3\times10^{-9}\times(0.02)} = 15.99\times10^6\ Pa = 15.99\ MPa.$ ↓

5/ • Determine the flexure and shear stresses point C.

$\sigma_c = \frac{-y}{c}\,\sigma_{max} = -\frac{7.5}{32.5}(-2.684) = 0.6194\ GPa = 619.4\ MPa.$ tension

$\tau_c = \frac{VQ}{It}$ $\quad Q = 2(10)(40)(30-17.5) = 10\times10^3\ mm^3 = 10\times10^{-6}\ m^3$

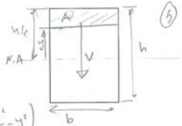
$\tau_c = \frac{11\times10^3\times10\times10^{-6}}{363.3\times10^{-9}\times0.02} = 15.14\times10^6\ Pa = 15.14\ MPa.$ ↓

III. [20 points]

15 • Derive the expression for the shear stress distribution in a rectangular section.

$$Q = \bar{y}A = \frac{\left(\frac{h}{2}+y\right)}{2}\left(\frac{h}{2}-y\right) b$$

$$= \frac{1}{2}\left(\frac{h^2}{4}-y^2\right) b$$



$$I = \frac{1}{12} bh^3 \quad (2)$$

$$\tau = \frac{V\left(\frac{1}{2}\right)\left(\frac{h^2}{4}-y^2\right)\cancel{b}}{\frac{1}{12} bh^3 (\cancel{b})} = \frac{6V}{bh^3}\left(\frac{h^2}{4}-y^2\right)$$

(1)

(2)

5 • Draw the distribution on the figure.

$$\tau_{max} = \frac{6V}{bh^3}\left(\frac{h^2}{4}-0\right)$$

$$= \frac{3V}{2bh} = \frac{3}{2}\frac{V}{A}$$